EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. A line $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z-k}{3}$ cuts the y-z plane and the x-y plane at A and B respectively. If $\angle AOB = \frac{\pi}{2}$, then find k, where O is the origin.

2. Find the volume of the tetrahedron with vertices P(2, 3, 2), Q(1, 1, 1), R(3, –2, 1) and S(7, 1, 4).

3. A sphere has an equation $|\vec{r}-\vec{a}|^2+|\vec{r}-\vec{b}|^2 = 72$ where $\vec{a}=\hat{i}+3\hat{j}-6\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}+2\hat{k}$. Find

(i) the centre of the sphere

(ii) the radius of the sphere

(iii) perpendicular distance from the centre of the sphere to the plane $\vec{r}=(2\hat{i}+2\hat{j}-\hat{k}) = -3$.

4. Find the equation of the sphere which is tangential to the plane x – 2y – 2z = 7 at (3, –1, –1) and passes through the point (1, 1, –3).

5. Let PM be the perpendicular from the point P(1, 2, 3) to the x-y plane. If OP makes an angle θ with the positive direction of the z-axis and OM makes an angle ϕ with the positive direction of the x-axis, where O is the origin, then find θ and ϕ.

6. Prove that the line $\frac{x}{1}=\frac{y}{1}=\frac{z-1}{-2}$ lies in the plane x + y + z = 1. Find the lines in the plane through the point (0, 0, 1) which are inclined at an angle $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$ with the line.

7. Find the equations of the straight line passing through the point (1, 2, 3) to intersect the straight line x + 1 = 2 (y – 2) = z + 4 and parallel to the plane x + 5y + 4z = 0.

8. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle of $\frac{\pi}{3}$.

9. Find the distance of the point P(–2, 3, –4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane 4x + 12y – 3z + 1 = 0.

10. Find the equation to the line passing through the point (1, –2, –3) parallel to the line
2x + 3y – 3z + 2 = 0 = 3x – 4y + 2z – 4.

11. Find the equation of the line passing through the point (4, –14, 4) and intersecting the line of intersection of the planes 3x + 2y – z = 5 and
x – 2y – 2z = –1 at right angles.

12. Let P = (1, 0, –1); Q = (1, 1, 1) and R = (2, 1, 3) are three points.
(a) Find the area of the triangle having P, Q and R as its vertices.

(b) Given the equation of the plane through P, Q and R in the form ax + by + cz = 1.

(c) Where does the plane in part (b) intersect the y–axis.

(d) Give parametric equations for the line through R that is perpendicular to the plane in part (b).

13. Find the point where the line of intersection of the planes x – 2y + z = 1 and x + 2y – 2z = 5, intersect the plane 2x + 2y + z + 6 = 0.

14. Feet of the perpendicular drawn from the point P(2, 3, –5) on the axes of coordinates are A, B and C. Find the equation of the plane passing through their feet and the area of ΔABC.

15. Find the equation to the line which can be drawn from the point (2, –1, 3) perpendicular to the lines

$\frac{x-1}{2}=\frac{y-2}{2}=\frac{z-3}{2}$ and $\frac{x-4}{3}=\frac{y}{2}=\frac{z+3}{1}$

16. Find the equation of the plane containing the straight line $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z}{5}$ and perpendicular to the plane x – y + z + 2 = 0.

17. Find the value of p so that the lines $\frac{x-1}{-3}=\frac{y-p}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ are in the same plane. For this value of p, find the coordinates of their point of intersection and the equation of the plane containing them.

18. Find the equations to the line of greatest slope through the point (7, 2, –1) in the plane
x – 2y + 3z = 0 assuming that the axes are so placed that the plane 2x + 3y – 4z = 0 is horizontal.

19. The line $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). Find the equation of the remaining sides.

20. Find the equation of the line which is reflection of the line $\frac{x-1}{9}=\frac{y-2}{-1}=\frac{z+3}{-3}$ in the plane
3x – 3y + 10z = 26.

21. Find the equation of the plane containing the line $\frac{x-1}{2}=\frac{y}{3}=\frac{z}{2}$ and parallel to the line $\frac{x-3}{2}=\frac{y}{5}=\frac{z-2}{4}$.
Find the also the S.D. between the two lines.